**Syllabus for AP® Calculus BC**

**2015-2016 School Year**

This course is intended to be an extension of Calculus AB rather than an enhancement. Topics from the AB course will be reviewed and elaborated. The calculus will also be applied to parametric, polar, and vector functions. Additional topics include slope fields, numerical methods for solving differential equations, the definition of limit, sequences, and series. The cost of the AP exam is approximately $87.

**Pre-requisites:** Teacher recommendation, AP Calculus AB, Graphing TI-83 or 84 calculator is required. Students have the choice of taking the AP test in May or taking the final exam (AP format) for the course. Students must make the decision in writing by October 1.

**Textbook**

Stewart, James. Single Variable Calculus: Concepts and Contexts. Belmont, CA: Brookes/Cole.

**Supplemental Resources**

*Calculus of a Single Variable.* Larson, Hostetler, and Edwards. 7th edition. Houghton Mifflin. Boston, MA: 2002.

*Exploring Calculus with the Geometers Sketchpad*. Clements, Pantozzi, Steketee. Key Curriculum Press. Emeryville, CA: 2002.

*Calculus Explorations*. Forester. Key Curriculum Press. Emeryville, CA: 1998.

*Be Prepared for the AP Calculus Exam.* Howell, M. and Montgomery M. Skylight Publishing. Andover, MA: 2005.

*Calculus Calculator Labs*, Benita Albert and Phyllis Hillis, 2000

 **Method of Instruction**

Emphasis will be placed on the rule of four: numerical, graphical, analytical, and verbal. All effort will be given to examine problems in all four ways and to make connections between the methods. Students are encouraged to discover and investigate as much as possible; emphasis is on discovery rather than lecture. During class, students are encouraged to show their solutions on the board and to work in groups, where appropriate. Communication is a major goal of the course; students are expected to communicate verbally and in written form, using appropriate vocabulary and well-written sentences. All assignments require students to justify their solution using multiple representations. Tests and quizzes will be a combination of calculator and non-calculator, with the majority being no calculators (as it is on the AP® exam). On these assignments also, students are required to justify their solutions using multiple representations.

**Use of Technology**

Students are required to have their own graphing calculator and to bring it to class on a daily basis. The TI-84+ will be the one used primarily. Graphing calculators will be used frequently to make connections among the rule of four and also to make computations with the derivative and integral. Additionally, through graphing calculator explorations, students will interpret results and verify solutions. Guided explorations using graphing calculators are integrated into the course. The explorations serve either to reinforce a concept or as a means of discovering a new concept. . Students use the graphing calculator to experiment with the zoom feature to discover the concept of local linearity and to confirm results that were first obtained analytically, such as the locations of extrema and inflection points. [SC12, 13] Students work in teams to complete the explorations. Students adapt quickly to using their calculators in clever ways, experimenting to find which line is tangent to a curve at a given point, evaluating limits based on graphical displays, and confirming results that were obtained analytically, such as the numerical values of derivatives or definite integrals. The instruction “support graphically” has become an often-used tool because it is so quick to do and the graph the calculator produces gives students confidence in their conjectures and answers. [SC13]

Where appropriate, students will use the computer software Geogebra to discover relationships and to verify results. As an additional resource, students will be able and expected to access class notes, worksheets, and outside links on our class website. Students use the graphing calculator to experiment with the zoom feature to discover the concept of local linearity and to confirm results that were first obtained analytically, such as the locations of extrema and inflection points.

**Grading Policy**

The students’ overall grade will be comprised mostly of grades on tests and quizzes. Homework will only be a small part of the grade. Students are required to maintain a notebook with class notes, tests, and other handouts. Students will solve problems not only from the textbook and from previously released AP® exams, but also students will solve problems from real-life sources, other texts, supplemental materials, and online resources.

**Topic Outline**

The following is the topic outline for the class. All aspects of the course outline as listed in the AP® Course Description will be covered; however, the sequence of topics is subject to change.

**Topic 1: Limits and Their Properties (12 Days)**

Content and/or Skills Taught:
- Review properties of functions in context
- Graphical interpretation of limits
- Numerical calculation of limits
- One-sided limits
- Properties of limits
- Existence of limits
- Infinite limits
- Limits at Infinity
- Continuity
- Types of discontinuities
- Intermediate Value Theorem

 **Topic 2: Differentiation (30 Days)**

Content and/or Skills Taught:
- Limit Definition of the Derivative
- Finding Equations of Tangent Lines
- Relating the derivative to the slope of the tangent line
- Estimating derivatives from tables and graphs
- Interpretation of the derivative as an instantaneous rate of change
- Graphical interpretation of differentiability / non-differentiability
- Differentiability implies continuity, but continuity does not necessarily imply differentiability.
- Rules for differentiation: Power, Product, Quotient, Chain
- Properties of derivatives
- Implicit differentiation
- Related rates
- Recognizing the need for the derivative in applied problems
- Differentiating exponential, logarithmic, and inverse trigonometric functions
- L'Hopital's Rule and Relative Rates of Growth (Exponential, Polynomial, Logarithmic)
- Finding the derivative of the inverse of a function at a point
- Differentiation of parametric and polar curves

 **Topic 3: Applications of Derivatives (30 Days)**

Content and/or Skills Taught:
- Extreme value theorem
- Finding absolute extrema of a function on a closed interval
- Increasing and decreasing functions
- Analyzing monotonicity of functions
- Interpreting the derivative in elementary physics applications, including position, velocity, and acceleration
- Rectilinear motion
- Finding local extrema by the first and second derivative tests
- Rolle's Theorem, The Mean Value Theorem, and their geometric interpretations
- Analyzing the relationship between f, f', and f'' by use of sign charts
- Determine intervals of concavity and points of inflection
- Curve sketching using all of the above information
- Using local linear approximations to estimate the value of a function from the tangent line equation
- Examine error bound from a local linear approximation
- Optimization of functions (local and global) as applied to real-world problems
- Applications of optimization to marginal business applications

 **Topic 4: Introduction to Integration and Differential Equations (30 Days)**

Content and/or Skills Taught:
- Antiderivatives as related to derivatives of elementary functions
- Rectilinear motion problems requiring antidifferentiation
- Visual interpretation of the family of curves that satisfy a differential equation via slope fields
- Finding specific solutions to a differential equation using an initial condition
- Review of sigma notation and properties of sigma notation
- Examining the definite integral as a Riemann sum using left sums, right sums, midpoint sums, and trapezoidal sums
- Develop the definite integral as a limit of these Riemann sums
- Properties of definite integrals
- First and second Fundamental Theorems of Calculus
- Average value of a function from an analytical and graphical approach
- Integrating rates of change to give the net change in the function over an arbitrary interval
- The Fundamental Theorems of Calculus as accumulation functions, both as accumulated area and in the context of applied problems
- Develop the formula for the solution to a differential equation given an initial condition by using the integral as an accumulation function
- Integration by u-substitution
- Solving separable differential equations, emphasizing exponential growth and decay, interest problems, and Newton's Law of Cooling

* ***Newton’s Law of Cooling Exploration using the TI-84 [SC12]***

 **Topic 5: Applications of Integration (10 Days)**

Content and/or Skills Taught:
- Finding the area of a region
- Finding the area of a region between two curves
- Volumes of solids of revolution (disk, washer methods)
- Volumes of solids with known cross sections
- Arc length of functions of the form y = f(x), x = g(y), and parametrically defined curves
- If time, shell method for finding volumes of solids of revolution, as well as surface area of such solids

 **Topic 6: Techniques of Antidifferentiation (10 Days)**

Content and/or Skills Taught:
- Integration by parts
- Integration by partial fractions
- Solving logistic differential equations
- Improper integrals

* ***Exploration: Improper Integration and Its Role in “Normalizing”, Albert and Hillis, Calculus Calculator Labs [SC12,13]***

***Objective: To utilize the calculator to apply an improper integration model to normal distribution***

- Continue discussion on slope fields and differential equations using Euler's Method

 **Topic 7: Polynomial Approximations and Series (25 Days)**

Content and/or Skills Taught:
- Review of Sequences, Monotonicity, Convergence of Sequences through analytical and graphical approach
- Limit definition of convergence of a series as a limit of a sequence of partial sums
- Tests for determining the convergence/divergence of a series of constants through the acronym PARTING C:
- P-series
- Alternating series test (with error bound)
- Ratio and Root Tests
- Telescoping Series
- Integral Test for convergence: The relationship between the convergence of a series with its associated definite integral. Show how this proves the P-series test.
- Nth Term Test for Divergence
- Geometric Series: Examining the ratio to determine convergence/divergence
- Comparison Test: Direct Comparison and Limit Comparison Tests
- The Harmonic and Alternating Harmonic Series:

- ***Investigated by TI-84 program that sums both series for an indefinite period***

- Introducing Power Series through Geometric Series: (For example, 1/(1-x))
- Using series tests to find intervals of convergence and testing the endpoints.
- Functions defined by power series through functional replacement, multiples of known power series, term-by-term differentiation, and term-by-term integration
- Graphically determining the interval of convergence and using an error function to determine the accuracy of the approximation
- Develop Taylor and Maclaurin series through local linear, local quadratic, local cubic and higher-degree polynomial approximations
- Emphasize that a Taylor polynomial is a Maclaurin polynomial translated by "a" units horizontally
- Require students to memorize Maclaurin series for e^x, sin(x), cos(x), and 1/(1-x) in both closed and expanded form
- Lagrange error bound for Taylor polynomials to calculate the degree of the polynomial needed for a desired accuracy

 **Topic 8: Polar Areas, Areas of Conic Sections, and Trigonometric Substitutions (8 Days)**

Content and/or Skills Taught:
- Determining the intersection of polar curves analytically and graphically
- Area bounded by one or more polar curves
- Trigonometric Substitutions
- Determining the area bounded by conic sections